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Seat No.	
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[5558]-101

**F.E. EXAMINATION, 2019
ENGINEERING MATHS—I
(2015 PATTERN)**

Time : Two Hours

Maximum Marks : 50

- N.B. :—** (i) Neat diagrams must be drawn wherever necessary.
(ii) Figures to the right indicate full marks.
(iii) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.
(iv) Assume suitable data, if necessary.
(v) Attempt Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6, Q. No. 7 or 8.

1. (a) Reduce the following matrix to its normal form and hence find its rank : [4]

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}.$$

- (b) Find eigen values and eigen vector corresponding to highest eigen value of the following matrix : [4]

$$A = \begin{bmatrix} -2 & -8 & -12 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

- (c) Solve the equation $x^3 - 1 = 0$ by applying Demoivre's theorem. [4]

P.T.O.

Or

2. (a) Investigate for what values of k the system of equations $x + y + z = 1, 2x + y + 4z = k, 4x + y + 10z = k^2$ have infinite number of solutions. [4]
- (b) Find locus of z such that : [4]
- $$|z + 1| = |z - i|.$$
- (c) If $\sin(x + iy) = u + iv$ prove that $u^2 \operatorname{cosec}^2 x - v^2 \sec^2 x = 1$ and $u^2 \sec^2 y + v^2 \operatorname{cosec}^2 y = 1$. [4]
3. (a) Solve any one : [4]

(i) Test for convergence the series $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$.

(ii) Test the convergence of the series :

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots$$

(b) Expand $\log(1 + x + x^2 + x^3)$ upto the term in x^8 . [4]

(c) Find the n th derivative of $y = \frac{x}{(x-1)(x-2)(x-3)}$. [4]

Or

4. (a) Solve any one : [4]
- (i) Evaluate :

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{\log(1+x) - x}.$$

(ii) Evaluate :

$$\lim_{x \rightarrow 0} (\sin x)^{\tan x}.$$

(b) Expand $x^3 + 7x^2 + x - 6$ in powers of $x - 3$. [4]

(c) If : [4]

$$y = e^{\tan^{-1} x},$$

prove that :

$$(1 + x^2)y_{n+2} + [2(n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0.$$

5. Solve any two :

(a) If $z = \tan(y + ax) + (y - ax)^{3/2}$, find the value of $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}$. [6]

(b) If $x^2 = au + bv$ and $y^2 = au - bv$, then prove that : [6]

$$\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \left(\frac{\partial v}{\partial y}\right)_x \cdot \left(\frac{\partial y}{\partial v}\right)_u.$$

(c) If $u = \sin^{-1}(x^2 + y^2)^{1/5}$, then prove that : [7]

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{2}{5} \tan u \left[\frac{2}{5} \tan^2 u - \frac{3}{5} \right].$$

Or

6. Solve any two :

(a) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, then prove that : [7]

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + 2f = 0.$$

(b) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$, then prove that : [6]

$$u_{xx} + u_{yy} = f''(r) + \frac{1}{r} f'(r).$$

- (c) If $z = f(x, y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, then prove that : [6]

$$y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}.$$

7. (a) If : [4]

$$x = uv, y = \frac{u + v}{u - v}$$

find :

$$\frac{\partial(u, v)}{\partial(x, y)}.$$

- (b) If $u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3$ find $\frac{\partial x}{\partial u}$. [4]

- (c) Divide the number 120 into three parts so that the sum of their products taken two at a time shall be maximum. [5]

Or

8. (a) Examine for functional dependence and independence $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + xz$. [4]

- (b) Find the percentage error in the area of an ellipse with an error of 1% is made in measuring its major and minor axis. [4]

- (c) Find the extreme values of $f(x, y) = x^3 + y^3 - 3axy, a > 0$. [5]